

## A NOTE ON POSITIVE DEFINITE MATRICES

In multivariate analyses, you might get an error message that indicates the matrix being manipulated is “not positive definite.” This can occur for a number of reasons.

1. Generalized least squares estimation requires the covariance matrix analyzed to be positive definite – maximum likelihood estimation is also problematic.
2. Some procedures create intermediate weight matrices during the process of calculation that may be not positive definite, creating problems for the continuation of computation.
3. In some cases, the model-based variance-covariance matrix may be not positive definite; LISREL will quit if this occurs.

A matrix is positive definite if all of its eigenvalues are positive.

The determinant can also be used to assess matrices for positive definite status. In symmetric matrices (i.e., sigma), it will be positive definite if each principal submatrix has a positive determinant. If the determinant of the matrix is zero, the matrix is singular, and GLS estimation methods requiring the inversion of the matrix will not proceed.

Since one interpretation of the determinant is it being a measure of generalized variance, a sigma or R matrix that fails to have a positive determinant is troubling because negative variances are undefined and zero variances only apply to constants.

Linear dependency may result in a matrix that is not positive definite. Perfectly correlated variables should be eliminated. Sometimes small samples result in sampling variation that create problems for solutions. Similarly, large amounts of missing data can have similar results.

When correlation matrices are computed with pairwise deletion, it creates correlations one at a time – since the estimation is not simultaneous, it can create a matrix that is not positive definite. Polychoric correlations are often estimated this way.

Correlations created with dichotomous scores can have odd properties that result in not positive definite matrices. Polychoric correlations often resolve these problems.