

LINEAR EQUATIONS KEY

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What is the determinant of $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$? $|\mathbf{A}| = \underline{(2)(3) - (1)(1) = 5}$

Does this system of equations have a unique solution? YES

What is the determinant of $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$? $|\mathbf{A}| = \underline{(2)(1) - (2)(1) = 0}$

Does this system of equations have a unique solution? NO

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Find the determinant and inverse:

1. $\mathbf{A} = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$ $\det(\mathbf{A}) = 12 - 12 = 0$, no inverse

2. $\mathbf{B} = \begin{bmatrix} 2 & 6 \\ 3 & 9 \end{bmatrix}$ $\det(\mathbf{B}) = 18 - 18 = 0$, no inverse

3. $\mathbf{C} = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$ $\det(\mathbf{C}) = 4 - 6 = -2$
 $\text{inv}(\mathbf{C}) = -\frac{1}{2} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -1/2 & 3/2 \\ 1 & -2 \end{bmatrix}$

4. $\mathbf{D} = \begin{bmatrix} 2 & -1 \\ 3 & 3 \end{bmatrix}$ $\det(\mathbf{D}) = 6 - (-3) = 9$
 $\text{inv}(\mathbf{D}) = 1/9 \begin{bmatrix} 3 & 1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 1/3 & 1/9 \\ -1/3 & 2/9 \end{bmatrix}$

Find the determinant:

5. $\mathbf{E} = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 3 & 4 \\ 5 & 2 & 7 \end{bmatrix}$ $\det(\mathbf{E}) = 42 - 20 + 18 - 45 - 16 + 21 = 0$

6. $\mathbf{F} = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 3 & 4 \\ 1 & 2 & 2 \end{bmatrix}$ $\det(\mathbf{F}) = 12 - 4 + 18 - 9 - 16 + 6 = 7$