Vectors KEY

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Identify each as a row vector, column vector, or scalar.

$(a_1 + a_2 + a_3 + a_4)$	Scalar
$\begin{bmatrix} 520 \\ 640 \\ 780 \end{bmatrix}$	3-element column vector
13	Scalar
(115, 129, 92, 89)	4-element row vector
5×11	Scalar
(0, 0, 0)	3-element row vector

Indicate whether each is equal or not equal.

$\begin{bmatrix} 5 \\ 6 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \end{bmatrix}$	Not equal
$\begin{bmatrix} 9 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 9 \\ 0 \\ 1 \end{bmatrix}$	Equal
$\begin{bmatrix} 55 \\ 69 \\ 48 \end{bmatrix}$ (55, 69, 48)	Not equal

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1. Find $\underline{z} = \underline{x} + \underline{y} = \begin{bmatrix} 56 \\ 64 \\ 32 \\ 88 \\ 90 \\ 79 \end{bmatrix} + \begin{bmatrix} 50 \\ 69 \\ 51 \\ 98 \\ 87 \\ 70 \end{bmatrix} = \begin{bmatrix} 106 \\ 133 \\ 83 \\ 186 \\ 177 \\ 149 \end{bmatrix}$

2. Find ½ $\underline{z}$ = [½] $\begin{bmatrix} 106 \\ 133 \\ 83 \\ 186 \\ 177 \\ 149 \end{bmatrix} = \begin{bmatrix} 53.0 \\ 66.5 \\ 41.5 \\ 93.0 \\ 88.5 \\ 74.5 \end{bmatrix}$

3. If the students took three exams, how would you write the computation to obtain the mean in vector notation?

Sum the three score vectors: $\underline{x} + \underline{y} + \underline{z} = \underline{t}$, then multiply by 1/3: 1/3 $\underline{t}$

4. If the mean score on the final was 60, find each student's deviation score employing vector notation.

$\underline{y} - \underline{60} = \underline{d}$

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$\underline{a}'\underline{b} = (1, 2, 4) \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} = (1 \cdot 2) + (2 \cdot 1) + (-1 \cdot 4) = 2 + 2 - 4 = 0$

$\underline{a} = \begin{bmatrix} 5 \\ 2 \\ 3 \end{bmatrix}$ compute $\underline{a}'\underline{a} = (5\ \ 2\ \ 3) \begin{bmatrix} 5 \\ 2 \\ 3 \end{bmatrix} = (5 \cdot 5) + (2 \cdot 2) + (3 \cdot 3) = 25 + 4 + 9 = 38$

$\underline{a} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$, $\underline{b} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ compute $\underline{a}'\underline{b}$ and $\underline{b}'\underline{a}$.

$\underline{a}'\underline{b} = (2\ \ 1\ \ 0) \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = 2 + 3 + 0 = 5$

$\underline{b}'\underline{a} = (1\ \ 3\ \ 2) \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = 2 + 3 + 0 = 5$

$\underline{a} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$, $\underline{s} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ compute $\underline{a}'\underline{s} = (2 \;\; 1 \;\; -1) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 2 + 1 + -1 = 2$

The Rule: the scalar product of any vector and a vector of ones yields the sum of the elements of the original vector.

$\underline{a}'\underline{e}_1 = 2 + 0 + 0 = 2$

$\underline{a}'\underline{e}_2 = 0 + 1 + 0 = 1$

$\underline{a}'\underline{e}_3 = 0 + 0 + -1 = -1$

$\underline{a} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$, $\underline{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, where $\underline{a}'\underline{0} = 0 + 0 + 0 = 0$

The Rule: the scalar product of any vector and a vector of zeros yields the scalar zero.