3. Parameter Estimation & Hypothesis Testing EPSY 8268

**Generalized least squares (GLS) estimates of the level-2 coefficients, γ*qs***

The basic Level-1 model is , where *rij* ~ N(0, σ2).







The basic Level-2 model is , where *u*0*j* ~ N(0, τ00).



 = Δ*j*

() = 



**Empirical Bayes (''EB'') estimates of randomly varying level-1 coefficients, β*qj***

The Bayes estimate of β*0j* is 

The weight λ*j* is the reliability of  as an estimate of β*0j*.

 = .

**Maximum likelihood estimates of variance and covariance components, σ2 at Level-1, and T at Level-2**

Because of the unbalanced nature of the data in most applications of hierarchical linear models (i.e., *ni* varies across the ***J*** units and the observed patterns on the level-1 predictors also vary), traditional methods for variance-covariance component estimation fail to yield efficient estimates. Through iterative computing techniques, such as the EM algorithm and Fisher scoring, maximum-likelihood estimates for σ2 and **T** can be obtained. In the program output, these estimates are denoted by σ2 and τ respectively.

We have been assuming that the variance and covariance components have been known. In practice, we need to estimate these. One way to think about maximum likelihood estimation is to find estimates of σ2, **T**, and γ that maximize the likelihood of observing the actual data Y.

**Some other useful statistics**

1. Reliability of .

Reliability for each *q* = 0, …, Q.

2. Least squares residuals, ().

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3. Empirical Bayes residuals ().

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**Hypothesis Testing**

*Hypothesis Tests for the Level-2 Fixed Effects and the Variance-Covariance Components*

|  |  |  |
| --- | --- | --- |
| Type of Hypothesis | Test Statistic | Program Output |
| Fixed level-2 effects |  |  |
| Single-parameter:  *H*0: γ*qs* = 0  *H*1: γ*qs* ≠ 0 | *t*-ratio (a) | Standard feature of the fixed effects table for all level-2 coefficients. |
| Multi-parameter (contrast vectors):  *H*0: ***C*′ γ** = **0**  *H*1: ***C*′ γ** ≠ **0** | General linear hypothesis test (Wald test), chi-square test (b) | Optional output specification |
| *Variance-covariance components* |  |  |
| Single-parameter:  *H*0: τ*qq* = 0  *H*1: τ*qq* ≠ 0 | Chi-square test (c) | Standard feature of the variance components table for all level-2 random effects. |
| Multi-parameter:  *H*0: **T** = **T**0  *H*1: **T** = **T**1 | Difference in deviances, likelihood ratio test (d), where H = D0 – D1 | Optional output specification |

(a) See Equation 3.83 in RB (2002).

(b) See Equation 3.91 in RB (2002).

(c) See Equation 3.103 in RB (2002).

(d) Here **T**0 is a reduced form of **T**1. See Equation 3.107 in RB (2002).

There are several advantages of multiparameter tests:

1. an omnibus test of the relationship between a categorical Level-2 predictor (e.g., a set of dummy variables for region) and a Level-1 parameter.
2. contrasts between categories of a Level-2 predictor.
3. examining whether a Level-2 characteristic interacts with any of several Level-1 predictors.
4. examining whether some subset of Level-2 predictors is needed in a particular β*qj* model.
5. many of these protect against increasing the probability of Type-I errors.