# 12. Cross-Classified Models EPSY 8268

**Cross-Classification**

Many cases exist where the units of observations belong to multiple higher-level units simultaneously. For example, students live in neighborhoods and attend schools, or students may have two different teachers simultaneously. Another example includes patients working with doctors and hospitals where hospitals take referrals from multiple doctors and doctors refer patients to various hospitals. Such models allow for a variety of analysis goals:

* Partition variance among students, schools, and neighborhoods
* Investigate the roles of characteristics of neighborhoods and schools
* Investigate the extent to which the associations between student characteristics and outcomes vary over neighborhoods and schools
* Estimate the unique effects of neighborhoods and schools

Level-1 Model *within cell*

 *Yijk* = π0*jk* + *eijk*where *eijk* ~ N(0, σ2)

A model for outcome *y* for case *i* nested within row *j* and column *k* of a two-way classification.

π0*jk* is the expected value of *Yijk* within cell *jk* (neighborhood × school). In this case *eijk* is the unique child effect, the deviation of the score for child *ijk* from the cell mean, and σ2 is the within cell variance.

Level-2 Models *between cell*

π0*jk* = θ0 + *b*00*j* + *c*00*k* where *b*00*j* ~ N(0, τ*b*00) and *c*00*k* ~ N(0, τ*c*00)

θ0 is the grand mean of *Y*

*b*00*j* is the neighborhood *j* effect, averaged over all schools

*c*00*k* is the school *k* effect, averaged over all neighborhoods

Neighborhood by School Crosstabulation (a sample of Schools & Neighborhoods)

|  |  |  |
| --- | --- | --- |
| Neighborhood ID | School ID | Total |
| 0 | 3 | 5 | 8 | 15 | 16 | 17 | 18 | 19 | 20 |
|  | 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 8 | 9 |
| 32 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 5 | 6 |
| 38 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 4 | 5 |
| 39 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 2 | 3 |
| 40 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 4 |
| 47 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 6 | 8 |
| 48 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 4 | 5 |
| 53 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 0 | 1 | 4 |
| 54 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 3 | 4 |
| 67 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 4 | 5 |
| 68 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 4 | 6 |
| 69 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 6 | 7 |
| 189 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 2 |
| 190 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 6 | 0 | 7 |
| 191 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 0 | 3 |
| 192 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 0 | 5 |
| 193 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 0 | 4 |
| 194 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 0 | 4 |
| 195 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 7 | 0 | 9 |
| 197 | 0 | 0 | 0 | 0 | 2 | 0 | 1 | 2 | 0 | 0 | 5 |
| 216 | 0 | 0 | 0 | 0 | 4 | 1 | 6 | 1 | 0 | 0 | 12 |
| 229 | 0 | 0 | 0 | 0 | 1 | 0 | 8 | 2 | 1 | 0 | 12 |
| 263 | 0 | 0 | 0 | 0 | 14 | 0 | 1 | 1 | 0 | 0 | 16 |
| 710 | 1 | 10 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 13 |
| 711 | 0 | 5 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 6 |
| 712 | 0 | 9 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 11 |
| 713 | 0 | 7 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 8 |
| … |  |  |  |  |  |  |  |  |  |  |  |
| Total | 146 | 159 | 155 | 112 | 190 | 111 | 154 | 91 | 102 | 174 | 2310 |

Example:

Garner and Raudenbush (1991) studied a sample of 2310 students from 524 neighborhoods (row factor) attending 17 schools (column factor) in Scotland. The outcome variable is a composite measure of academic achievement (total attainment) across a series of national exams.

Data input into HLM includes a level-1 file (within cell) with student information, a level-2 row-factor file with neighborhood information, and a level-2 column-factor file with school-level information. The level-1 within-cell file includes IDs for both level-2 factors.

Level-1 File

Educational attainment; primary 7 verbal reasoning, primary 7 reading…



Level-2 Files

Social Deprivation (based on presence of 20 indicators of resources)

 

1. **Specify the level-1 within-cell model, the unconditional model for ATTAIN.**

**Level-1 Model**

    *ATTAINijk* = π0*jk* + *eijk*

**Level-2 Model**

    π0*jk* = θ0 + *b*00*j* + *c*00*k*

**Final estimation of fixed effects:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Fixed Effect |  Coefficient |  Standard error |  *t*-ratio |  Approx. *d.f.* |  *p*-value |
| For INTRCPT1, *π0* |
|      INTERCEPT,θ*0* | 0.075355 | 0.072224 | 1.043 | 1769 | 0.297 |

**Final estimation of row and level-1 variance components:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Random Effect | Standard Deviation | Variance Component |   *d.f.* | χ2 | *p*-value |
| INTRCPT1/ ICPTROW,*b00j* | 0.37566 | 0.14112 | 523 | 904.87451 | <0.001 |
| level-1, *e* | 0.89390 | 0.79906 |   |   |   |

**Final estimation of column level variance components:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Random Effect | Standard Deviation | Variance Component |   *d.f.* | χ2 | *p*-value |
| INTRCPT1/ ICPTCOL,*c00k* | 0.27469 | 0.07545 | 16 | 120.42539 | <0.001 |

Intra-neighborhood correlation (dependency in outcomes for students within the same neighborhood but attend different schools):

$$ICC\left(neighborhood, row\right)=\frac{τ\_{b00}}{τ\_{b00}+τ\_{c00}+σ^{2}}=\frac{0.141}{0.141+0.075+0.799}=0.139$$

Intra-school correlation (dependency in outcomes for students who attend the same school but live in different neighborhoods):

$$ICC\left(school, column\right)=\frac{τ\_{c00}}{τ\_{b00}+τ\_{c00}+σ^{2}}=\frac{0.075}{0.141+0.075+0.799}=0.074$$

Intra-cell correlation (dependency in outcomes of students who live in same neighborhood and attend the same school):

$$ICC\left(within-cell\right)=\frac{τ\_{b00}+τ\_{c00}}{τ\_{b00}+τ\_{c00}+σ^{2}}=\frac{0.141+0.075}{0.141+0.075+0.799}=0.212$$

1. **Complete the level-1 within-cell model.**

**Level-1 Model**

    *ATTAINijk* = π0*jk* + π1*jk*\*(*P7VRQijk*) + π2*jk*\*(*P7READijk*) + π3*jk*\*(*DADOCCijk*) + π4*jk*\*(*DADUNEMPijk*) + π5*jk*\*(*DADEDijk*) + π6*jk*\*(*MOMEDijk*) + π7*jk*\*(*MALEijk*) + *eijk*

**Level-2 Model**

    π0*jk* = θ0 + *b*00*j* + *c*00*k*
    π1*jk* = θ1
    π2*jk* = θ2
    π3*jk* = θ3
    π4*jk* = θ4
    π5*jk* = θ5
    π6*jk* = θ6
    π7*jk* = θ7

Because of the sparseness of the data and the potential complexity of the model, the effects of the level-1 explanatory variables are fixed.

1. **Build a level-2 row-factor model. In this model, the association between social deprivation and educational attainment is fixed.**

**Level-1 Model**

    *ATTAINijk* = π0*jk* + π1*jk*\*(*P7VRQijk*) + π2*jk*\*(*P7READijk*) + π3*jk*\*(*DADOCCijk*) + π4*jk*\*(*DADUNEMPijk*) + π5*jk*\*(*DADEDijk*) + π6*jk*\*(*MOMEDijk*) + π7*jk*\*(*MALEijk*) + *eijk*

**Level-2 Model**

    π0*jk* = θ0 + *b*00*j* + *c*00*k* + (γ01)\**DEPRIVEj*
    π1*jk* = θ1
    π2*jk* = θ2
    π3*jk* = θ3
    π4*jk* = θ4
    π5*jk* = θ5
    π6*jk* = θ6
    π7*jk* = θ7

**Final estimation of fixed effects:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Fixed Effect |  Coefficient |  Standarderror |  *t*-ratio |  Approx.*d.f.* |  *p*-value |
| For INTRCPT1, *π0* |
|      INTERCEPT,θ*0* | 0.094514 | 0.021185 | 4.461 | 1769 | <0.001 |
|      DEPRIVE, *γ01* | -0.155997 | 0.025680 | -6.075 | 522 | <0.001 |
| For P7VRQ, *π1* |
|      INTERCEPT,θ*1* | 0.027565 | 0.002263 | 12.180 | 1769 | <0.001 |
| For P7READ, *π2* |
|      INTERCEPT,θ*2* | 0.026237 | 0.001750 | 14.992 | 1769 | <0.001 |
| For DADOCC, *π3* |
|      INTERCEPT,θ*3* | 0.008100 | 0.001361 | 5.953 | 1769 | <0.001 |
| For DADUNEMP, *π4* |
|      INTERCEPT,θ*4* | -0.120682 | 0.046779 | -2.580 | 1769 | 0.010 |
| For DADED, *π5* |
|      INTERCEPT,θ*5* | 0.143454 | 0.040789 | 3.517 | 1769 | <0.001 |
| For MOMED, *π6* |
|      INTERCEPT,θ*6* | 0.059497 | 0.037381 | 1.592 | 1769 | 0.112 |
| For MALE, *π7* |
|      INTERCEPT,θ*7* | -0.055941 | 0.028390 | -1.970 | 1769 | 0.049 |

**Final estimation of row and level-1 variance components:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Random Effect | Standard Deviation | Variance Component |   *d.f.* | χ2 | *p*-value |
| INTRCPT1/ ICPTROW,*b00j* | 0.06593 | 0.00435 | 522 | 554.29863 | 0.159 |
| level-1, *e* | 0.67446 | 0.45489 |   |   |   |

**Final estimation of column level variance components:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Random Effect | Standard Deviation | Variance Component |   *d.f.* | χ2 | *p*-value |
| INTRCPT1/ ICPTCOL,*c00k* | 0.06151 | 0.00378 | 15 | 35.39235 | 0.002 |

The residual variation within cell was reduced from 0.799 to 0.455 and the variance between neighborhoods and schools were substantially reduced (are now near zero).

*A model with a random effect of Social Deprivation*

Above, the association between neighborhood social deprivation and attainment was assumed invariant across schools. We can test this assumption by allowing this effect to randomly vary (include *c*01*k*). A model comparison test based on the deviances produces a nonsignificant result – there is no evidence that the association between neighborhood social deprivation and attainment varies over schools.

In the next model, we include a row-specific predictor random (neighborhood deprivation) and a column-specific predictor random (school dummy variable). This model includes all of the student-level explanatory variables and their associated coefficients, π*pjk*. θ*p* is the intercept, the expected value of π*pjk* (when all explanatory variables are zero). βs are the fixed effects for column-specific predictors; *b*s are the random effects associated with the column-specific predictors; γs are the fixed effects for row-specific predictors; *c*s are the random effects associated with the row-specific predictors

**Level-1 Model**

    *ATTAINijk* = π0*jk* + π1*jk*\*(*P7VRQijk*) + π2*jk*\*(*P7READijk*) + π3*jk*\*(*DADOCCijk*) + π4*jk*\*(*DADUNEMPijk*) + π5*jk*\*(*DADEDijk*) + π6*jk*\*(*MOMEDijk*) + π7*jk*\*(*MALEijk*) + *eijk*

**Level-2 Model**

-    π0*jk* = θ0 + *b*00*j* + *c*00*k* + (γ01 + *c*01*k*)\**DEPRIVEj* + (β01 + *b*01*j*)\**SchoolDUMMYk*
    π1*jk* = θ1
    π2*jk* = θ2
    π3*jk* = θ3
    π4*jk* = θ4
    π5*jk* = θ5
    π6*jk* = θ6
    π7*jk* = θ7

**Final estimation of fixed effects:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Fixed Effect |  Coefficient |  Standard error |  *t*-ratio |  Approx. *d.f.* |  *p*-value |
| For INTRCPT1, *π0* |
|      INTERCEPT,θ*0* | 0.148873 | 0.047311 | 3.147 | 1228 | 0.002 |
|      DEPRIVE, *γ01* | -0.158655 | 0.026366 | -6.017 | 521 | <0.001 |
|      DUMMY, *β02* | -0.023178 | 0.017512 | -1.324 | 14 | 0.207 |
| For P7VRQ, *π1* |
|      INTERCEPT,θ*1* | 0.027609 | 0.002261 | 12.210 | 1228 | <0.001 |
| For P7READ, *π2* |
|      INTERCEPT,θ*2* | 0.026239 | 0.001749 | 15.001 | 1228 | <0.001 |
| For DADOCC, *π3* |
|      INTERCEPT,θ*3* | 0.008132 | 0.001360 | 5.979 | 1228 | <0.001 |
| For DADUNEMP, *π4* |
|      INTERCEPT,θ*4* | -0.121035 | 0.046741 | -2.589 | 1228 | 0.010 |
| For DADED, *π5* |
|      INTERCEPT,θ*5* | 0.142689 | 0.040732 | 3.503 | 1228 | <0.001 |
| For MOMED, *π6* |
|      INTERCEPT,θ*6* | 0.061618 | 0.037343 | 1.650 | 1228 | 0.099 |
| For MALE, *π7* |
|      INTERCEPT,θ*7* | -0.054709 | 0.028383 | -1.928 | 1228 | 0.054 |

**Final estimation of row and level-1 variance components:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Random Effect | Standard Deviation | Variance Component |   *d.f.* | χ2 | *p*-value |
| INTRCPT1/ ICPTROW,*b00j* | 0.12591 | 0.01585 | 116 | 125.05766 | 0.266 |
| INTRCPT1/ DUMMY,*b01j* | 0.03123 | 0.00098 | 116 | 122.68775 | 0.317 |
| level-1, *e* | 0.67418 | 0.45452 |   |   |   |

Note: The chi-square statistics reported above are based on only 119 of 524 units that had sufficient data for computation. Fixed effects and variance components are based on all the data.

**Final estimation of column level variance components:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Random Effect | Standard Deviation | Variance Component |   *d.f.* | χ2 | *p*-value |
| INTRCPT1/ ICPTCOL,*c00k* | 0.05328 | 0.00284 | 14 | 27.60991 | 0.016 |
| INTRCPT1/ DEPRIVE,*c01k* | 0.02108 | 0.00044 | 14 | 9.66845 | >0.500 |

**Three-Level Cross-Classified Models**

It is possible to estimate a cross-classified model where either the rows or the column are nested in a higher level factor. For example, repeated measures over time (level-1) can be cross-classified by students (rows) and teachers (columns) and teachers can be nested within schools (clusters), where students can change schools (as they change teachers).

Source: Raudenbush, S.W. & Bryk*,* A.S. (2002). *Hierarchical Linear Models. Applications and Data Analysis Methods* (2nd ed., pp. 373-398). Sage Publications.