A little statistical theory regarding the treatment of items or dichotomously scored variables

A single item k is administered to individual i which results in score X_{ki}

The mean of an item k scored on individuals i = 1 to N:

$$\frac{\sum_{i=1}^{N} X_{ki}}{N} = \mu_k$$; Where X_k is a single item = 0 or 1 , $\mu_k = p_k = \bar{X}_k$

The variance of an item:

Recall $\sigma_k^2 = \frac{\sum_{i=1}^{N} (X_{ki} - \bar{X}_k)^2}{N}$ is the variance of item k (any given item), scored on individual i

Through some algebraic manipulation, we can derive:

$$\sigma_k^2 = \frac{\sum_{i=1}^{N} (X_{ki} - p_k)^2}{N} = \frac{\sum X_{ki}^2}{N} - \frac{2\sum p_k X_{ki}}{N} + \frac{\sum p_k^2}{N}$$

Now we can replace these terms:

$$\frac{\sum X_k^2}{N} = \frac{\sum X_k}{N} = \bar{X}_k = \mu_k = p_k$$ where X = 0 or 1, $\mu_k = p_k$

to arrive at:

$$\sigma_k^2 = p_k - 2p_k^2 + p_k^2 = p_k - p_k^2 = p_k(1 - p_k) = p_k q_k$$

p is the proportion of examinees that answered the item correctly;
(1-p) = q is the proportion of examinees that answered the item incorrectly.